Classification of Bridge Methods of Measuring Impedances *

By JOHN G. FERGUSON

An analysis is made of the requirements for satisfactory operation of the simple four-arm bridge when used for impedance measurements. The various forms of bridge are classified into two major types called the ratio-arm type and the product-arm type, based on the location of the fixed impedance arms in the bridge. These two types are subdivided further, based on the phase relation which exists between the fixed arm impedances. Eight practical forms of bridges are given, three of them being duplicate forms from the standpoint of the method of measuring impedance. These bridges together allow the measurement of any type of impedance in terms of practically any type of adjustable standard. The use of partial substitution methods and of resonance methods with these bridges is discussed and several methods of operation are described which show their flexibility in the measurement of impedance.

Introduction

BRIDGE methods have been used for the measurement of impedance from the very beginning of alternating current use. In fact, the history of the impedance bridge dates back to the earlier bridges developed for the measurement of direct current resistance. While some objection may be raised to this method of measurement on the count that it is not direct indicating, in the sense that an ammeter or voltmeter is, this has been more than offset by the high accuracy of which it is capable. Bridge methods of measuring impedance have accordingly continued to hold a high place in the field of electrical measurements and except perhaps at the higher radio frequencies are considered supreme for this purpose over the whole frequency range, where high accuracy is the principal requirement.

The peculiar advantages of the bridge method are most evident where emphasis is laid on the circuit characteristics rather than on power requirements. In power engineering it may be more logical to make measurements in terms of current, voltage, and power, since these are the quantities of immediate interest. In communication engineering, however, where design is based for the most part on circuit characteristics, and power considerations are only of secondary interest, it is natural that bridge methods, which furnish a direct comparison of these circuit characteristics should be generally preferred.

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Due to the wide field of usefulness and great flexibility of the impedance bridge, a very large amount of development work has been done and a considerable amount of literature has been published covering various types and modifications. In fact, the subject has become so broad and the information so voluminous that the engineer who has not specialized in the subject has every excuse for a feeling of considerable confusion when he finds it necessary to make a choice among the numerous circuits available. Perhaps the greatest single obstacle to a still more extensive use of the impedance bridge in industry is this very multiplicity of types combined with a rather complete lack of any practical guide for the engineer who is interested principally in the measurement itself and looks on the bridge simply as a means to this end.

Very little information is available as to the relative merits of the various types of bridges, the great majority of published articles being confined to a description of a particular circuit used by the author for a particular purpose.

The present article furnishes a comparison of the relative merits of the large number of circuits which are available for making the same measurement and should serve as a guide to the engineer who is more interested in results than in acquiring a broad education in bridge measurements. An outline is given of the fundamental requirements which must be met by bridges used for impedance measurements, and a classification is made which serves as a help in the choice of a bridge for any particular type of measurement. The relative merits of the simpler types of bridge are discussed from the standpoint of the measurement of both components of an impedance, particularly with reference to measurements in the communication range of frequencies from about 100 to 1,000,000 cycles. Where only the major component of an impedance is desired, for instance where only the inductance of a coil or the capacitance of a condenser is desired, the requirements are not so severe and many forms of bridges may be used which are not suitable for the purpose here outlined. Bridges are also used to a large extent for other purposes than impedance measurements, such as for frequency measurements. These applications will not be considered here.

The General Bridge Network

Any bridge may be considered as a network consisting of a number of impedances which may be so adjusted that when a potential difference is applied at two junction points, the potential across two other junction points will be zero. For this condition, there are relations

between certain of the impedances which enable us to evaluate one of them in terms of the others. Thus the bridge is essentially a method of comparing impedances. The impedances of the bridge may consist of resistance, capacitance, self and mutual inductance, in any combinations, and they may actually form a much more complicated network than the simple circuit shown in Fig. 1. Consequently, the number of

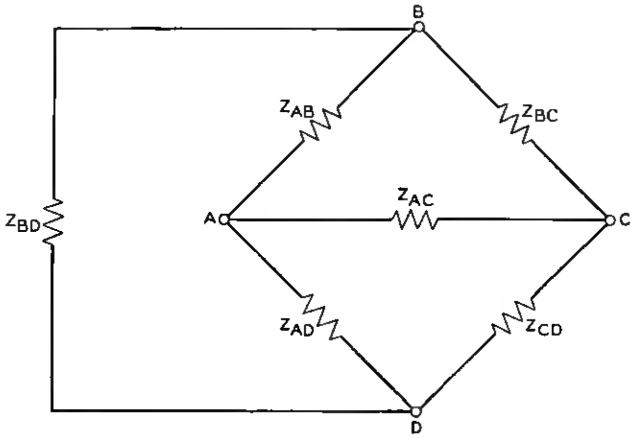

Fig. 1—Schematic of the impedance bridge reduced to its simplest form.

different bridges which can be devised for the measurement of impedances is extremely large. However, since only four junction points are significant, any bridge circuit may be reduced to a network of six impedances connected between these four points, as shown in Fig. 1. These impedances are direct impedances, that is, there are no mutual impedances between them.

If a potential is applied at BD and the balance condition is that the potential be zero across AC, then the points BD are called the input or power source terminals and the points AC are called the output or detector terminals. The impedances Z_{BD} and Z_{AC} then act simply as shunts across the power source and detector respectively and do not affect the balance relation. The balance is not affected if the power source and detector are interchanged in a bridge reduced to this simple form and hereafter no distinction will be made in this respect.

After the bridge has been reduced to the form of Fig. 1, the equation for balance is

$$Z_{CD}Z_{AB} = Z_{BC}Z_{AD},$$

from which

$$Z_{CD} = \frac{Z_{BC}Z_{AD}}{Z_{AB}}. \tag{1}$$

Thus, if Z_{CD} is the unknown impedance, equation (1) evaluates it in terms of the other three impedances. Equation (1) is a vector equation and therefore the value of Z_{CD} both in magnitude and phase, or both components of it when considered as a complex quantity, may be obtained from this equation.

Although the above equations and subsequent discussion are based primarily on the use of impedances, it should be remembered that all of these relations may be obtained in the same general form if the bridge arms are considered as admittances.

The Bridge Requirements

If the impedances of equation (1) are replaced by the complex equivalents $R + jX$, then

$$R_{CD} + jX_{CD} = \frac{(R_{BC} + jX_{BC})(R_{AD} + jX_{AD})}{R_{AB} + jX_{AB}}. \tag{2}$$

From this equation R_{CD} and X_{CD} may be evaluated in terms of the other six quantities. Thus, if each component of the impedances of three arms is known, each component of the fourth impedance in terms of the other six components can be determined.

In obtaining the balance, any or all of the six component impedances occurring in the right hand side of equation (2) may be adjusted. Since there are two unknown quantities to be determined, at least two of these components must be adjusted. From the standpoint of simplicity and speed in operation and in order to keep the cost of the circuit to a minimum, it is desirable that not more than two of the known components be adjustable. It is also essential that the choice be such that a variation of one adjustable standard balance one component of the unknown, irrespective of the other component. In other words R_{CD} should be balanced by one known standard, this value of the standard being independent of the magnitude of X_{CD}, and, in turn, X_{CD} should be balanced by another standard, the value of which should be independent of the magnitude of R_{CD}. This condition of independent adjustment for the two components is essential for satisfactory operation of the bridge, since it allows the balance to be made more rapidly and systematically, and a given setting of one standard always corresponds to the same value of one component of the unknown, independent of the magnitude of the other component, thus allowing the calibration of each of the adjustable standards in terms of the unknown component which it measures.

To meet this requirement, the two components for use as adjustable standards should be so chosen that, when equation (2) is reduced to

the general form

$$R_{CD} + jX_{CD} = A + jB, \tag{3}$$

where A and B are real quantities, one of the adjustable impedances will appear in A and not in B, while the other will appear in B but not in A.

Consideration of equation (2) shows that if adjustable standards consisting either of both components of Z_{BC} or of both components of Z_{AD}, are chosen, and if the impedances of the two remaining arms are selected so that their ratio is either real or imaginary, but not complex, then equation (2) reduces to the form of equation (3). No other combination will meet the requirement taking equation (2) as it stands. Since for the general case there is no essential difference in the resulting type of bridge whether Z_{AD} or Z_{BC} is used as our adjustable standard, this means that there is really only one method of adjustment, namely the use of both components of one adjacent impedance.

However, if it is realized that parallel components may be used instead of series components for the standard, then equation (2) may be rewritten as follows:

$$R_{CD} + jX_{CD} = (R_{AD} + jX_{AD})\ (R_{BC} + jX_{BC})(G_{AB} - jB_{AB}) \tag{4}$$

where

$$G_{AB} - jB_{AB} = Y_{AB} = \frac{1}{Z_{AB}} \cdot$$

From this it follows that G_{AB} and B_{AB} may be used as the adjustable standards, by making the product $Z_{AD}Z_{BC}$ real or imaginary.

Thus there are two methods of adjustment possible, either the two series components of an adjacent arm or the two parallel components of the opposite arm.

Having chosen the adjustable standards, there remain in each case two arms, adjacent in one case and opposite in the other, which have fixed values. These impedances must meet certain definite requirements, as already stated.

For the case of adjustment by an adjacent arm, that is, by Z_{AD}, equation (2) may be written in the form

$$R_{CD} + jX_{CD} = \frac{Z_{BC}}{Z_{AB}}(R_{AD} + jX_{AD}). \tag{5}$$

Then in order that this equation fulfill the requirements expressed by equation (3), the vector ratio of the fixed arms must be either real or

imaginary but not complex, that is, the difference between their phase angles must be 0°, 180° or ± 90°.

For the case of adjustment by the opposite arm Z_{AB}, equation (4) may be written in the form

$$R_{CD} + jX_{CD} = Z_{BC}Z_{AD}(G_{AB} - jB_{AB}). \tag{6}$$

Then in order that this equation fulfill the requirements of equation (3), the vector product of the fixed arms must be either real or imaginary, but not complex, that is, the sum of their phase angles must be 0°, 180° or ± 90°.

In the case of bridges of the type indicated by equation (5), the fixed arms always enter the balance equation as a ratio, and are therefore called ratio arms, the bridges of this type being called ratio arm bridges.

In the case of bridges of the type indicated by equation (6), the fixed arms always enter the balance equation as a product, and are therefore called product arms, the bridges of this type being called product arm bridges.

These two types may be further subdivided according to whether the term involving the fixed arms is real or imaginary.

It should be pointed out at this time that the fixed arms are fixed in value only to the extent that they are not varied during the course of a measurement. They may be functions of frequency, and may be arbitrarily adjustable to vary the range of the bridge, but they are not adjusted in the course of balancing the bridge.

Classification of Bridge Types

The foregoing discussion shows that all simple four arm bridges meeting the requirements specified may be divided into four types. The balance equations of these four types may now be simply derived from the general equations (2) and (4).

1. *Ratio Arm Type—Ratio Real*

If Z_{BC}/Z_{AB} is real, then

$$\theta = \theta_{BC} - \theta_{AB} = 0° \text{ or } 180°.$$

That is

$$Z_{BC}/Z_{AB} = R_{BC}/R_{AB} = X_{BC}/X_{AB}. \tag{7}$$

Substituting equation (7) in equation (5) and separating,

$$R_{CD} = \frac{R_{AD}R_{BC}}{R_{AB}} = \frac{R_{AD}X_{BC}}{X_{AB}} \tag{8}$$

and

$$X_{CD} = \frac{X_{AD}R_{BC}}{R_{AB}} = \frac{X_{AD}X_{BC}}{X_{AB}}. \tag{9}$$

For this type it follows from equations (8) and (9) that the components of Z_{CD} are balanced by components of Z_{AD} of the same phase, that is R_{AD} will balance R_{CD}, and X_{AD} will balance X_{CD}.

2. *Ratio Arm Type—Ratio Imaginary*

If Z_{BC}/Z_{AB} is imaginary, then

$$\theta = \theta_{BC} - \theta_{AB} = \pm 90°.$$

That is

$$Z_{BC}/Z_{AB} = jX_{BC}/R_{AB} = -jR_{BC}/X_{AB}. \tag{10}$$

Substituting equation (10) in equation (5) and separating,

$$R_{CD} = -\frac{X_{AD}X_{BC}}{R_{AB}} = \frac{X_{AD}R_{BC}}{X_{AB}} \tag{11}$$

and

$$X_{CD} = \frac{R_{AD}X_{BC}}{R_{AB}} = -\frac{R_{AD}R_{BC}}{X_{AB}}. \tag{12}$$

For this type it follows from equations (11) and (12) that the components of Z_{CD} are balanced by components of Z_{AD} 90° out of phase, that is X_{AD} will balance R_{CD} and R_{AD} will balance X_{CD}.

3. *Product Arm Type—Product Real*

If $(Z_{BC}Z_{AD})$ is real, then

$$\theta = \theta_{BC} + \theta_{AD} = 0° \text{ or } 180°.$$

That is

$$Z_{BC}Z_{AD} = Z_{BC}/Y_{AD} = R_{BC}/G_{AD} = -X_{BC}/B_{AD}. \tag{13}$$

Substituting equation (13) in equation (6)

$$R_{CD} = \frac{G_{AB}R_{BC}}{G_{AD}} = -\frac{G_{AB}X_{BC}}{B_{AD}} \tag{14}$$

and

$$X_{CD} = -\frac{B_{AB}R_{BC}}{G_{AD}} = \frac{B_{AB}X_{BC}}{B_{AD}}. \tag{15}$$

For this type the components of Z_{CD} are balanced by components of Y_{AB} of the same phase, that is G_{AB} will balance R_{CD} and B_{AB} will balance X_{CD}.

4. *Product Arm Type—Product Imaginary*

If $(Z_{BC}Z_{AD})$ is imaginary, then

$$\theta = \theta_{BC} + \theta_{AD} = \pm 90°.$$

That is

$$Z_{BC}Z_{AD} = Z_{BC}/Y_{AD} = jR_{BC}/B_{AD} = jX_{BC}/G_{AD}. \tag{16}$$

Substituting equation (16) in equation (6)

$$R_{CD} = \frac{B_{AB}R_{BC}}{B_{AD}} = \frac{B_{AB}X_{BC}}{G_{AD}} \tag{17}$$

and

$$X_{CD} = \frac{G_{AB}R_{BC}}{B_{AD}} = \frac{G_{AB}X_{BC}}{G_{AD}}. \tag{18}$$

For this type the components of Z_{CD} are balanced by components of Y_{AB} 90° out of phase, that is B_{AB} will balance R_{CD} and G_{AB} will balance X_{CD}.

The relations given in these equations are summarized in Table I.

TABLE I

BRIDGE TYPES

Unknown	Adjustable Standard			
	Ratio Arm Type		Product Arm Type	
	Ratio Real	Ratio Imaginary	Product Real	Product Imaginary
R_{CD}	R_{AD}	X_{AD}	G_{AB}	B_{AB}
X_{CD}	X_{AD}	R_{AD}	B_{AB}	G_{AB}
G_{CD} [1]	G_{AD}	B_{AD}	R_{AB}	X_{AB}
B_{CD} [1]	B_{AD}	G_{AD}	X_{AB}	R_{AB}

[1] These values may be derived by using admittances in place of impedances and vice versa throughout.

ACTUAL BRIDGE FORMS

The fixed arms may be made up of simple resistances or reactances or of complex impedances provided they meet their phase requirements. Since the choice of complex impedances has no practical advantages over simple reactances or resistances, the choice of fixed impedances should obviously be made on the basis of the simplest practical type. So they will be limited for the present to simple resistance, capacitance, and self inductance.

Fig. 2 gives all of the combinations of fixed arms which meet the phase angle requirements already stated, when limited to simple resistance, inductance, or capacitance. For all forms, the magnitude of one arm is given in terms of the other and of a constant K, such

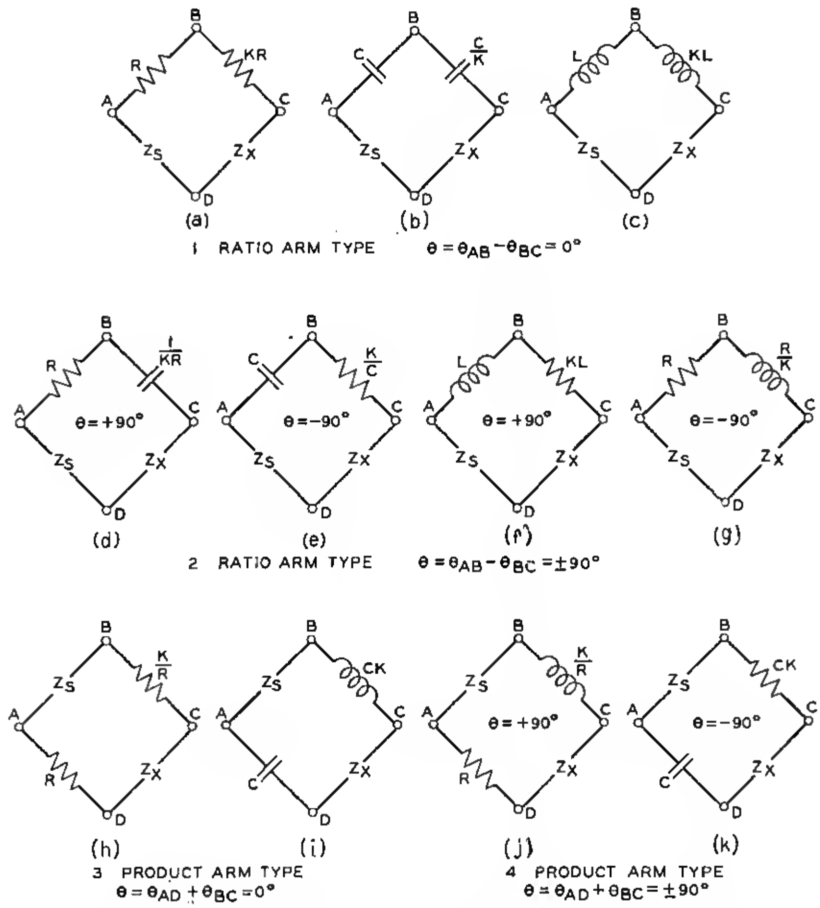

Fig. 2—The various forms of 4-arm bridges divided into four types. Forms f, g and j are impractical.

that the only term which appears in the balance equation is the term K. None of these bridges represents a distinctly new type, but since the classification is by means of the fixed impedance arms, one of them may be used to measure several types of impedance. Accordingly, it may correspond to more than one of the well-known bridge types.

For this reason, any references to, or comparison with existing special types of bridge are omitted.

TABLE II
BALANCE EQUATIONS

Unknown	Ratio Arm Type			Product Arm Type		
	$\theta = 0$	$\theta = +90°$	$\theta = -90°$	$\theta = 0$	$\theta = +90°$	$\theta = -90°$
$R_{CD} =$	KR_{AD}	KL_{AD}	K/C_{AD}	KG_{AB}	K/L'_{AB}	KC'_{AB}
$L_{CD} =$	KL_{AD}	—	KR_{AD}	KC'_{AB}	KG_{AB}	—
$C_{CD} =$	KC_{AD}	$1/KR_{AD}$	—	KL'_{AB}	—	$1/KG_{AB}$
$G_{CD} =$	KG_{AD}	$1/KL'_{AD}$	C'_{AD}/K	R_{AB}/K	L_{AB}/K	$1/KC_{AB}$
$L'_{CD} =$	KL'_{AD}	—	K/G_{AD}	C_{AB}/K	K/R_{AB}	—
$C'_{CD} =$	KC'_{AD}	G_{AD}/K	—	L_{AB}/K	—	R_{AB}/K
Figures....	2A 2B 2C	2D 2F[2]	2E 2G[2]	2H 2I	2J[2]	2K

[2] These forms are not practical.

R, L and C = series components of complex arms.

G, L' and C' = parallel components of complex arms.

K has the value indicated on the individual circuits of Fig. 2.

$$\theta = \theta_{AB} - \theta_{BC} \qquad \text{for Ratio Arm Type}$$

$$\theta = \theta_{AD} + \theta_{BC} \qquad \text{for Product Arm Type}$$

Table II gives the balance equations for each type of bridge for the measurement of any component of the unknown impedance in terms of resistance, capacitance, and inductance. These equations are simply derived from the general equations (8) to (18) by substitution of circuit constants for impedances and by the introduction of the constant K. This constant must be evaluated from the relation between the ratio arms or product arms shown in the individual bridge forms of Fig. 2. At the bottom of Table II are given the corresponding bridge figures for reference. This table shows no bridges having a phase relation of 180° between the fixed arms. A little consideration will show that since the phase relation between the unknown and the standard for such bridges must also be 180°, they cannot be used to measure any but pure reactances or negative resistances. Accordingly, they are not considered herein. In the case of the 90° relation, both signs must be considered and result in bridges which are complimentary with respect to one another, that is while one measures only inductive impedances, the other measures only capacitive impedances. Thus

Table II shows the imaginary type subdivided into two subtypes, depending on the sign of the angle.

As an example of the use of this table: Suppose it is desired to measure the series resistance and inductance of an unknown impedance. This may be done by using adjustable standards of series resistance and inductance, series resistance and capacitance, parallel resistance and capacitance, or parallel resistance and inductance, by choosing the particular type of bridge for the purpose. For instance, referring to Table II, if it is desired to measure the series resistance in terms of conductance, and the series inductance in terms of parallel capacitance, the product arm bridge with real ratio, that is either Fig. 2*h* or 2*i*, would be used.

Since there are six types of balance equations given in Table II, it follows that five of the circuits of Fig. 2 are duplicates of others from the standpoint of the balance equations which they give. For instance, there is no difference whatever in the theoretical operation of the bridges of Figs. 2*a*, 2*b*, and 2*c*. The choice must be determined entirely from other considerations. In the same way, as indicated by the figures tabulated in Table II, Figs. 2*d* and 2*f* give identical results as do Figs. 2*e* and 2*g*, and Figs. 2*h* and 2*i*. From the practical standpoint, there may be, and actually there is, considerable difference in the merits of these different forms. At this time, we may simply state that where a choice is possible, resistance is the preferred form of fixed arm and capacitance is preferred to inductance. This allows us to choose our preferred forms as Fig. 2*a*, Fig. 2*d*, Fig. 2*e*, and Fig. 2*h*.

A study of Table II shows that bridges of fixed ratio arm type always measure the series components of the unknown in terms of series components of the standard and, conversely, they measure the parallel components in terms of parallel components of the standard. Bridges of product arm type measure the series component of the unknown in terms of parallel components of the standard and conversely.

None of the balance equations of Table II includes frequency, that is, all of them allow the evaluation of each component of the unknown directly in terms of a corresponding component of the standard with the exception that in some cases the relation is a reciprocal one. Practically any form of standard may be chosen in order to measure a given type of unknown impedance.

Practical Considerations

So far the question whether the requirements for the fixed arm impedances given in Fig. 2 can be met in practice has not been con-

sidered. It may be well to point out that the performance of the bridge is determined very much by the degree to which the phase angle requirements are met. If there is appreciable error here, the two balances will not be entirely independent and necessary corrections will be complicated and difficult to make. Consequently, the first essential for a satisfactory bridge is that its fixed arms meet their phase angle requirements. For a general purpose bridge these requirements must hold independent of frequency at least over an appreciable frequency range.

The forms given in Fig. 2 meet their phase angle requirements at all frequencies provided the arms are actually pure resistances or reactances. If they have residuals associated with them, it is still possible to meet the phase angle requirements in most cases, at least over a reasonable frequency range, as discussed below.

Resistances can be made to have practically zero phase angle, and condensers, particularly air condensers, may be made to have phase angles of practically 90°. In the case of condensers having dielectric loss, this loss may be kept quite small. However, it takes such a form that the phase difference of the condenser is approximately independent of frequency. For this reason, it can not be represented accurately either as a fixed resistance in series with the condenser or as a fixed conductance in shunt, when considered over a frequency range. Due to the small amount of this loss, it is usually satisfactory to represent it in either one form or the other, whichever is the more convenient.

In the case of inductance, there is always a quite appreciable series resistance which, for the usual size of coil, can not be neglected and must accordingly be corrected for.

With the above considerations in mind, the forms of Fig. 2 may now be reconsidered from the practical standpoint. It is readily seen that the requirements of the real ratio type bridge can be met using resistances, capacitances, or inductances. In the case of the imaginary ratio type, the requirements can be met, at least very approximately, in the case of Figs. 2*d* and 2*e*. However, in the case of Figs. 2*f* and 2*g*, any resistance in series with the inductance must be corrected by a capacitance in series with the resistance, if the correction is to be independent of frequency. Since the value of this series capacitance will, in general, be large, this form of correction is unsatisfactory. For instance, for a bridge in which the value of R is 1000 ohms and the inductance has a high time constant, the series capacitance required is in the order of 3 μf. By using a standard of inductance having larger series resistance, we may reduce this

capacitance, but we then have a form of bridge which is, in effect, a compromise between Figs. 2*f* and 2*g*, and Figs. 2*d* and 2*e*, which has no practical advantages over the latter. Accordingly, the forms of Figs. 2*f* and 2*g* must be considered impractical, particularly as Figs. 2*d* and 2*e* give identical performance.

In the case of the product arm type the requirements can be met by Fig. 2*h* and can be met by Fig. 2*i* by adding a conductance in shunt with the capacitance to compensate for the series resistance of the inductance. However, even though this allows us to meet the requirement, this form is less satisfactory than that of Fig. 2*h* due to the difficulty of designing an inductance standard having inductance and series resistance invariable over an appreciable frequency range. Again the requirements can be readily met by Fig. 2*k*, but in the case of Fig. 2*j* series resistance of the inductance can be corrected only by shunting the resistance arm by pure inductance, which is impractical. This is unfortunate since it rules out one form of bridge for which there is no duplicate and, consequently, makes the measurement of inductive impedances by bridges of this type impractical.

Summarizing the above, practical considerations rule out Figs. 2*f*, 2*g*, and 2*j*, reducing to five the number of different bridge types. There are eight forms remaining, namely three of the real ratio type, each capable of giving the same performance; two of the imaginary ratio type which are complementary, together giving a measurement of inductive and capacitive impedances; two of the real product type which will measure all types of impedance; and one imaginary product type which is capable of measuring only capacitive impedances.

The only duplicate forms are in the case of the real ratio and real product types. In the case of the latter, Fig. 2*h* is to be preferred in practically all cases to Fig. 2*i*, as already explained, and thus we can say that, practically speaking, we have duplicate forms only in the case of the real ratio type.

The three forms of this type are all used and each has certain advantages for certain types of measurements. This type of bridge, commonly known as the direct comparison type, is probably used more than any other, and is one of the most accurate types, particularly in the special case of equal ratio arms. This is due to the fact that a check for equality of the ratio arms may be readily made by a method of simple reversal without any external measurements, and by this means practically all the errors of the bridge may be eliminated. Resistance ratio arms are preferable for a general purpose bridge because they are more readily available and more readily adjusted to meet their requirements. They also give an impedance independent

of frequency, which is usually desirable. Capacitance ratio arms have certain advantages for particular cases. They may be readily chosen to give high impedance values, this being an advantage in certain cases, for instance in the measurement of small capacitances at low frequencies. This form is also desirable where high voltages must be used, since the ratio arms may be designed to withstand high voltages without the dissipation of appreciable energy. It also has the advantage that where measurements are desired with a direct current superimposed on the alternating current, the direct current is automatically excluded from the ratio arms and thus all of the direct current applied to the bridge passes through the unknown and there is no dissipation due to the direct current in the ratio arms. The impedance of the ratio arms decreases as the frequency increases, which is usually a disadvantage but may have advantages in some cases, such as the measurement of capacitance. There may be a disadvantage, in some cases, due to the load on the generator being capacitive, thus tending to increase the magnitude of the harmonics, and again, in the case of the measurement of inductances, there may be undesirable resonance effects.

The inductance ratio arm type has advantages where heavy currents must be passed through the bridge, since the ratio arms of this type may be designed to carry large currents with low dissipation. A modification of this type, where there is mutual inductance between the ratio arms, gives the advantage of ratio arms of high impedance with a corresponding low impedance input. A further modification consists in making the ratio arms the secondary of the input transformer, thus combining in one coil the functions of ratio arms and input transformer. This form, of course, departs from the simple four-arm bridge, but is mentioned here due to its simplicity and actual practical advantages.

Substitution Methods

In any of the bridges discussed and, in fact, in practically all bridges, it is possible to evaluate the unknown by first obtaining a balance with the unknown in the circuit and then substituting for it adjustable standards which may be adjusted to rebalance the bridge. This is, in general, a very accurate method, eliminating to a large degree the necessity for the bridge to meet its phase angle requirement. However, in the case of complete substitution of standards to balance both components of the unknown, the method has no advantage except accuracy over the bridges of type 1, Fig. 2, since standards of the same type as the unknown must be used and, in general, this method lacks

the flexibility of bridges of type 1, obtained by their unequal ratio arms. On the other hand, the use of substitution to measure the resistance or conductance component of the unknown has many advantages, the principal one being that it allows the choice of a type of bridge which will give directly the reactance component of the unknown in terms of an adjustable resistance and then by use of the substitution method to balance the resistance or conductance of the unknown by means of a second adjustable resistance, thus obtaining the ideal method of balance, using two adjustable resistances.

For the purpose of illustration, the case of the measurement of an inductive impedance may be taken. In general, the most desirable method would be to balance the reactance by means of series resistance. This can be done by means of the bridges of Figs. 2*e* or 2*g*. Choosing Fig. 2*e* as the preferred form, the bridge would normally take the form of Fig. 3*a*.

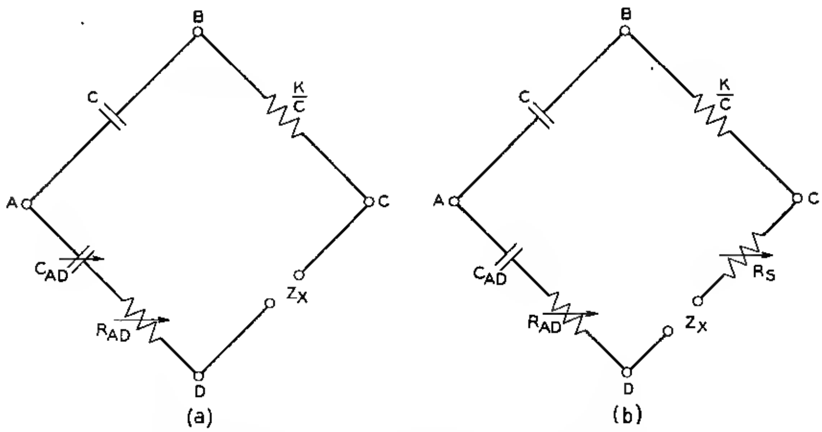

Fig. 3—(*a*) Bridge of type 2 for measuring self-inductance. (*b*) The same bridge modified by the use of partial substitution.

For normal operation, C_{AD} and R_{AD} would be the adjustable standards. The series inductance of the unknown would be given directly as KR_{AD}, while the series resistance would be given as K/C_{AD}. This measurement of the series resistance requires an adjustable capacitance and a computation due to the reciprocal relation. Now suppose a fixed value for C_{AD} were used and an adjustable resistance standard R_S placed in series with Z_X, giving the form of Fig. 3*b*, in which R_{AD} and R_S are the adjustable standards. If terminals Z_X are short circuited, the conditions for balance are $R_S = K/C_{AD}$ and

$R_{AD} = 0$. Then the unknown Z_X is inserted and the bridge rebalanced. The inductance of the unknown is given, as for Fig. 3*a*, as KR_{AD}, but since C_{AD} is unchanged the total resistance in CD is unchanged. Therefore, the series resistance of the unknown will be equal to the change in R_S between the two balances.

This bridge circuit may be recognized as the familiar bridge due to Owen,[3] and it is, theoretically at least, when used as described, an exceedingly desirable bridge for inductance measurements.

It should be pointed out here that since either C_{AD} or R_S may equally well be used to balance R_X, it is not necessary to use either one or the other exclusively in any one bridge. The adjustments may be combined so that the capacitance adjustment will take care of large changes and R_S of small changes; that is, C_{AD} may be used for coarse adjustment and R_S for fine adjustment. This compromise is, in general, more satisfactory than either method used alone.

The imaginary product arm type, particularly the form of Fig. 2*k*, is also well adapted to modification to enable it to measure capacitance and conductance in terms of two adjustable resistances.

There is a further modification of the substitution method, which is in common use. As already explained, there is little practical advantage in the substitution method for measuring either inductance or capacitance. However, there are occasions where the substitution of capacitance for inductance has advantages. Since the reactance of one is opposite in sign to that of the other, the method might more correctly be termed a compensation method, but in common with other substitution methods it can be made irrespective of the type of bridge. Various modifications of the general method may be used, but they are all classed under the general head of resonance methods.

Resonance Methods

If it is desired to measure the inductance of any inductive impedance, a capacitance standard may be inserted in series with it, and adjusted until the total reactance of the combination is zero. The only function the bridge performs is to measure the effective resistance of the combination and to determine the condition of zero reactance. Any of the bridges of Fig. 2 will do this satisfactorily, but those of real ratio type, that is the simple comparison type, are the most satisfactory since they give the resistance directly in terms of an adjustable resistance standard. This type of bridge is usually termed a series resonance bridge. The value of the inductance is computed from the resonance formula $\omega^2 LC = 1$. It has the dis-

[3] D. Owen, *Proc. Phys. Soc.*, London, October, 1914.

advantage that it involves the frequency, but it has the compensating advantage that the method, being essentially a direct measurement of the resistance of the resonant circuit, is very accurate for the measurement of effective resistance.

The condenser may equally well be shunted across the unknown, in which case the bridge circuit is called a parallel resonance bridge. However, if the ratio of reactance to resistance of the unknown is not high, the expression for the series inductance in this case is not as simple as that for series resonance, and is not independent of the value of the effective resistance, that is the two adjustments are not independent.

Fig. 4 shows the forms taken by the *CD* arm for resonance measurements. Fig. 4*a* is the series resonant circuit using an adjustable capacitance standard. Fig. 4*b* is the parallel circuit using an adjustable capacitance standard.

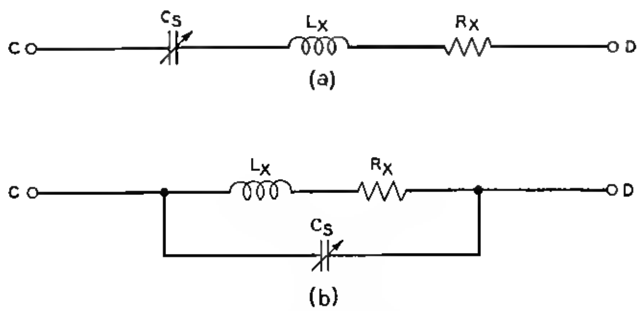

Fig. 4—(*a*) The CD arm of the bridge as used for series resonance measurements. (*b*) The CD arm of the bridge as used for parallel resonance measurements.